Indian Statistical Institute
Bangalore Centre
B.Math Third Year 2014-2015
Second Semester

Back Paper Examination Date : 28.05.15

Statistics IV

Answer as much as you can. The maximum you can score is 120.
The notation used have their usual meaning unless stated otherwise.
Time :- 3 hours

1. Consider the following three-way table. Take X = convict's race, Y = death penalty and Z = victim's race.

		Death Penalty		
Victim's Race	Defendant's Race	Yes	No	Percent Yes
White	White	53	414	11.3
	Black	11	37	22.9
Black	White	0	16	0.0
	Black	4	139	2.8
Total	White	53	430	11.0
	Black	15	176	7.9

(a) Write down the partial tables for different Z and also the marginal table.

(b) Find the estimates (say $\hat{P}$) of $P(Y = D|X = Q), Q = W, B$ and then find $\hat{P}(Y = D|X = B) - \hat{P}(Y = D|X = W)$ from all three tables.

(c) Explain why the values in partial tables are in the reverse direction of the marginal table. [6 + 3 x 2 + 6 = 18]

2. Suppose $X_1, \cdots X_n$ is a random samples from a continuous distribution and R_i is the ranks of X_i. Let $R = (R_1, \cdots R_n)'$.

(a) Show that R is uniformly distributed over the set of all permutations of the integers $\{1, \cdots n\}$.

(b) Find the following probabilities : (i) $P[R_i = r]$ and $P[R_i = r, R_j = s], i \neq j$.

(c) Show that

(i) $E[R_i] = n(n + 1)/2$.

(ii) $Var[R_i] = (n + 1)(n - 1)/12$.

(iii) $Cov[R_i, R_j] = -(N + 1)/12, i \neq j$.

[4 + (2 + 3) + (2 + 4 + 4) = 19]

3. (a) What is meant by (i) a nonrandomised and (ii) a randomised decision rule ? What is a risk function ? How is the loss function related to the risk function of a (i) nonrandomised and (ii) a randomised decision rule ?

   (b) Define (i) a minmax rule and (ii) a Bayes'rule w.r.t. a prior distribution π. What is the Bayes risk of a decision rule w.r.t. a prior distribution π?

   (c) Consider the problem of estimation of a real parameter θ with loss proportional to squared error.

   (i) Show that a Bayes' decision rule w.r.t. a prior is the mean of the corresponding posterior distribution of θ.

   (ii) Suppose a Bayes' decision rule δ_0 w.r.t. a prior π is also unbiased for θ. Then show that the Bayes risk of $\delta_0 = 0$.

   (d) Suppose X follows normal distribution with mean θ and variance 1. Consider a prior distribution π which is normal with mean 0 and variance σ^2.

   Show that the M.L.E δ_1 is not a Bayes' rule. Also show that the Bayes risk of δ_1 tends to the Bayes risk of δ_0 as σ tends to ∞.

   [(2 x 4 + 3) + (2 + 2 + 3) + (4 + 6) + (4 + 4) = 36]

4. Consider a subset S of a k-dimensional Euclidean space.

   (a) When is S said to be bounded from below ? What is the lower boundary $\lambda(S)$ of S ? When is it said to be closed from below ?

   (b) Suppose S is convex and is bounded from below. Show that the lower boundary of S is not empty.

   (c) Suppose Θ is finite the risk set $\mathcal{R}$ is bounded from below and closed from below. Consider the class of decision rules $D_0 = \{\delta : R(\delta) \in \lambda(\mathcal{R})\}$. [Here the ith co-ordinate of $R(\delta)$ is $R(\theta_i, \delta), \theta_i \in \Theta$]. Show that D_0 is a minimal complete class.

   [(2 + 3 + 1) + 8 + 13 = 27]